Correlating the Higgs and Kaon Sector CP Violations

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Abstract

Taking the most general two-doublet models with explicit CP violation in the Higgs sector but no phase in the CKM matrix, we determine the correlation between the Higgs and Kaon sectors using the experimental values of ΔM_K and ϵ_K. It is found that: there is a direct correlation between strength of FCNC couplings and the Higgs masses, and the lightest Higgs is nearly pure CP–even, but this small CP–odd composition gives large enough contribution to ϵ_K.

In the standard electroweak theory (SM), the single phase in the CKM matrix [1] is the unique source of CP violation which can appear only in the charged current interactions. The physics beyond the SM, however, introduces novel sources of CP violation which can arise in charged as well as neutral current vertices. Indeed, CP violation observables offer quite useful tools [2] in search for new physics in near future B factories such as LHC-B [3]. However, for a given model comprising the SM one has to saturate all observables either conserving or violating the CP symmetry.

In this letter, we assume that there is no CP violation in the SM, *i.e.*, CKM matrix is real. Furthermore, we take the most general two–doublet model with explicit CP violation. Then we attempt to answer the following question: **"Under such circumtances, what is the correlation between Higgs sector and (B^0, K^0)CP observables ?"** In fact, possibility of a real CKM matrix was discussed in [4] by taking into account all constraints from K^0 and B^0 systems: The present experimental constraints allow for a real CKM matrix provided that the physics beyond the SM contributes to both K^0 and B_d^0 systems. In particular, at least 20% of ΔM_{B_d} should follow from new physics contribution modulo other conditions coming from K^0 system. Unlike [4] which assumes spontaneous CP violation, we will restrict ourselves to explicit CP violation in the Higgs sector. In case of spontaneous CP violation which guarantees CP invariance of the Lagrangian before the electroweak

breaking, it is possible to keep CKM matrix real by introducing appropriate discrete symmetries [4]. However, with explicit CP violation in the Higgs sector one can not generate a real CKM matrix naturally. In such a case one can take reality of CKM matrix granted and analyze (K^0, B^0)CP observables with pure new physics CP violation coming from the Higgs sector only. In passing, one notes that in case of spontaneous CP violation electroweak baryogenesis constraints imply a light pseudoscalar Higgs [5]. In summary, in what follows we will assume that CKM matrix is pure real and all CP violation effects in the (K^0, B^0) system spring from the explicit CP violation in the Higgs potential.

We adopt a general two–doublet model [6] with most general CP–violating soft and hard operators:

$$
\begin{aligned}
V(\Phi_1, \Phi_2) &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \left[\mu_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.\right] \\
&+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1) \\
&+ \frac{1}{2} \left[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + h.c.\right] + \left[\left(\lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2\right) \Phi_1^\dagger \Phi_2 + h.c.\right]
\end{aligned}
\tag{1}
$$

where the dimensionless couplings $\lambda_{1,\cdots,4}$ are all real whereas $\lambda_{5,6,7}$ as well as the soft mass parameter μ_{12}^2 can have nontrivial phases. One can regard (1) as a direct extension of the SM Higgs sector to two Higgs doublets [6], or as a remnant of the supersymmetric models broken above the weak scale [7]. Altough our framework is completely nonsupersymmetric, to reduce the unknowns in a viable manner, we will take $\lambda_{1,\cdots,4}$ from the tree level minimal supersymmetric model [7]

$$
\lambda_1 = \lambda_2 = -\frac{1}{8}(g_1^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4}(g_1^2 - g_2^2), \quad \lambda_4 = -\frac{1}{2}g_1^2,
\tag{2}
$$

but vary $\lambda_{5,6,7}$ freely. As usual we expand the neutral components of the Higgs doublets around the electroweak vacuum as

$$
\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i + \phi_i + ia_i \end{pmatrix}, \quad (i = 1, 2)
\tag{3}
$$

where $v_1 = v \, \cos\beta$, $v_2 = v \, \sin\beta$ with $v = 246.2$ GeV for correct electroweak breaking.

We introduce the CP–even scalars h, H, the CP–odd scalar A, and the Goldstone boson G^0 (eaten by the Z boson in acquiring the mass) via the unitary rotation

$$
\begin{pmatrix} a_1 \\ a_2 \\ \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta & 0 & 0 \\ \sin\beta & -\cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} G_0 \\ A \\ h \\ H \end{pmatrix}
\tag{4}
$$

Then, in (H, h, A) basis the mass–squared matrix of the Higgs scalars assume the form:

$$
M^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 \\ M_{12}^2 & M_{22}^2 & M_{23}^2 \\ M_{13}^2 & M_{23}^2 & M_a^2 \end{pmatrix}
\tag{5}
$$

whose entries can be expressed in terms of the parameters of potential (1) as

$$
\begin{aligned}
M_{11} &= M_a^2 - (1/2)v^2 \sin^2 2\beta \left[\lambda_1 + \lambda_2 - (\lambda_3 + \lambda_4) \right] \\
&\quad - 2v^2 \left[\sin^3\beta \Re\{\lambda_{56}\} + \cos^3\beta \Re\{\lambda_{75}\} - (1/2)\sin 2\beta \Re\{\lambda_{76}\} \right]
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
M_{12} &= v^2 \sin 2\beta \left[-\lambda_1 \cos^2\beta + \lambda_2 \sin^2\beta + (1/2)\cos 2\beta (\lambda_3 + \lambda_4) \right] \\
&\quad + v^2 \sin 2\beta \left[\cos\beta \Re\{\lambda_{75}\} - \sin\beta \Re\{\lambda_{56}\} + \cot 2\beta \Re\{\lambda_{76}\} \right]
\end{aligned}
\tag{7}
$$

$$
M_{13} = v^2 \left[\cos\beta \Im\{\lambda_{75}\} - \sin\beta \Im\{\lambda_{56}\} \right]
\tag{8}
$$

$$
\begin{aligned}
M_{22} &= -2v^2 \left[\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + (1/4)\sin^2 2\beta (\lambda_3 + \lambda_4) \right] \\
&\quad - v^2 \sin 2\beta \left[\cos\beta \Re\{\lambda_{56}\} + \sin\beta \Re\{\lambda_{75}\} + \Re\{\lambda_{76}\} \right]
\end{aligned}
\tag{9}
$$

$$
M_{23} = -v^2 \left[\cos\beta \Im\{\lambda_{56}\} + \sin\beta \Im\{\lambda_{75}\} \right]
\tag{10}
$$

which depend on three new combinations of $\lambda_{5,6,7}$: $\lambda_{56} = \sin\beta\lambda_5 + \cos\beta\lambda_6$, $\lambda_{75} = \sin\beta\lambda_7 + \cos\beta\lambda_5$, $\lambda_{76} = \sin^2\beta\lambda_7 + \cos^2\beta\lambda_6$. It is understood that we have expressed the soft mass parameters μ_1^2, μ_2^2 and μ_{12}^2 in terms of the other parameters of the potential using the minimization condition. In the following, the Higgs mass square matrix (5) will be diagonalized numerically via $O^T \cdot M^2 \cdot O = diag. \left(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2 \right)$ where O is the orthonormal Higgs mixing matrix.

Perhaps the most spectacular property of the Higgs mass–squared matrix (5) is that the CP–even (H, h) and CP–odd (A) components are mixed [7,8]. The entries responsible for this mixing are M_{13}^2 and M_{23}^2 both of which depend on the imaginary parts of λ_{ij}. Hence, the explicit CP violation in the Higgs potential (1) shuffles the opposite CP components so that the mass eigenstate Higgs scalars have no longer definite CP quantum numbers. In the following,

this Higgs sector CP violation will be the unique source of CP violation in saturating the (K^0, B^0)CP observables through the exchange of neutral Higgs scalars inducing flavour–changing neutral currents (FCNC) at tree level.

In general, FCNC's induced by the neutral Higgs exchanges depend on the model employed for the Yukawa lagrangian [9]. On the other hand, in the case of spontaneous CP violation in the Higgs sector with nonvanishing phases of the Higgs doublets [4,10], the requirement of a real CKM matrix leads at once to the following mass matrices for quarks

$$\Gamma^u = \tan\beta \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{11}$$

$$\Gamma^d = \begin{pmatrix} m_d(\tan\beta - Sx^2) & -m_d S\beta_k & -m_d S\beta_d \\ -m_s S\beta_k & m_s(\tan\beta - Sy^2) & -m_s S\beta_s \\ -m_b S\beta_d & -m_b S\beta_s & m_b(\tan\beta - Sz^2) \end{pmatrix} \tag{12}$$

where $S = \tan\beta + \cot\beta$, and $\beta_k = xy$, $\beta_d = xz$, and $\beta_s = yz$ with $x^2 + y^2 + z^2 = 1$ determine the strengths fo FCNC transitions in K^0, B_d^0 and B_s^0 systems, respectively. This flavor structure forbids FCNC's among up-type quarks, that is, there is no D^0–$\overline{D^0}$ mixing. This particular flavor structure follows from the requirement of a real CKM matrix with spontaneous CP violation in the Higgs sector [10]. Instead of the parametrizations for a rather general two–doublet models [9], in the following we will stick to the Yukawa structures (11) and (12) as an *ansatze*. In a two–doublet model with explicit CP violation (1), such a Yukawa structure cannot be justified as in [4]; however, it will prove useful in analyzing (K^0, B^0)CP systems.

The detailed analysis in [4] shows that for saturating the present experimental bounds, at least 20% of ΔM_{B_d} must come from the new physics contribution. This requirement eventually boils down to $(\beta_d/\beta_K)^2 \sim 1$, which we will assume. Therefore, it is allowed to concentrate only on the K^0 system for the aim of saturating ΔM_K, ϵ_K, and observing the resulting constraints on the Higgs sector of the theory.

The effective $\Delta S = 2$ Hamiltonian for K^0–$\overline{K^0}$ system receives contributions from the exchange of charged bosons $W^\pm W^\pm$ (the SM contribution) , $W^\pm H^\pm$, $H^\pm H^\pm$ (two–doublet model contribution) as well as the neutral Higgs scalars h, H, A (two–doublet model with tree level FCNC). Since the CKM matrix is

real, only neutral Higgs bosons contribute to ϵ_K

$$\epsilon_K \equiv \frac{1}{\sqrt{2}} \frac{\Im \langle \overline{K^0}|M_{12}|K^0\rangle}{\Delta M_K} , \tag{13}$$

whereas the neutral kaon mass difference depends on the exchange of all bosons above,

$$\Delta M_K \approx 2|\Re \langle \overline{K^0}|M_{12}|K^0\rangle| . \tag{14}$$

The $W^\pm$ and $H^\pm$ contributions to $\langle \overline{K^0}|M_{12}|K^0\rangle$ can be found in [9,11], and the exchange of the neutral scalars contribute by (See, for instance, the first reference in [9], and [4])

$$\langle \overline{K^0}|M_{12}|K^0\rangle = \frac{G_F^2}{12\pi^2} f_K^2 M_K \frac{m_d}{m_s}(1 + \frac{m_d}{m_s})^{-1} \sum_k \left(\frac{2\sqrt{3}\pi v M_K}{M_{H_k}}\right)^2 (U_{k,12})^2 \tag{15}$$

where $U_{k,ij}$ is defined by

$$U_{k,ij} = -\frac{1}{2}(S_{k,ij} - S_{k,ji}^*) , \quad \text{with} \quad S_{k,ij} = -\frac{\Gamma_{ij}^d}{\sqrt{m_i m_j}}(O_{1k} + iO_{3k}) \tag{16}$$

using the convention that $k = 1, 2, 3$ counts the mass eigenstate neutral Higgs scalars whereas $i, j = 1, 2, 3$ are the generation indices.

Equipped with the necessary formulae for the Higgs and K^0 sectors, we now turn to a numerical tracing of the parameter space. We restrict the experimental constraints to the following bands:

$$0.98 \leq |\frac{\epsilon_K}{(\epsilon_K)^{exp}}| \leq 1.02 \quad \text{and} \quad 0.98 \leq |\frac{\Delta M_K}{(\Delta M_K)^{exp}}| \leq 1.02 \tag{17}$$

together with the positivity of the Higgs masses. In doing this, we wander in the parameter space varying $|\lambda_5|$, $|\lambda_6|$, $|\lambda_7|$, $\text{Arg}[\lambda_5]$, $\text{Arg}[\lambda_6]$, $\text{Arg}[\lambda_7]$, β_K, $\tan\beta$, and M_a freely.

We first analyze the Higgs sector within the experimental band of (17). Depicted in Fig. 1 is the scatter plot of such an analysis for H_1 (top), H_2 (middle) and H_3 (bottom). Each window here shows the variation of the CP composition (in %) of mass–eigenstate Higgs boson with its mass. It is clear that H_1 and H_2 are heavy Higgs bosons and they have approximately alternate CP properties: H_1 is composed mostly of CP even elements H (" $+$ ") and h (" $\times$ ")

though it has also considerable CP odd composition ("·") for $M_{H_1} \lesssim 500$ GeV. On the other hand, the other heavy Higgs H_2 has complementary properties compared to H_1 with a similar range of masses. The third Higgs boson, whose mass lies in 50 GeV $\lesssim M_{H_3} \lesssim 100$ GeV, is the lightest of all three and it has exclusively even CP, that is, its odd CP composition is below 0.1% in the entire parameter space. As is clear from (15), contribution of H_3 to $\langle \overline{K^0} | M_{12} | K^0 \rangle$ is large due to its relatively small mass; however, its CP–odd composition is also small.

Next, we show in Fig.2 the variation of β_K (which is already constrained to be $< 1/2$ [4]) with the absolute magnitudes of $\lambda_{5,6,7}$ for the special case of $|\lambda_5| = |\lambda_6| = |\lambda_7|$. The general tendency of the solution points is that the required value of β_K increases roughly linearly with λ. Since larger the λ larger the curvature of the potential, we conclude that large β_K requires large Higgs masses. This should be the case because $\langle \overline{K^0} | M_{12} | K^0 \rangle$ is proportional to β_K and one needs to suppress the FCNC due to neutral Higgs exchanges to agree with $(\Delta M_K)^{exp}$.

Depicted in Fig.3 is the dependence of M_a on $\tan \beta$. As the figure suggests, higher the value of $\tan \beta$ lower the value of M_a. This, in particular, implies that K^0 constraints do not allow the Higgs sector be in the decoupling regime in which the heavy Higgs bosons become degenerate and the lightest Higgs mass becomes maximal (~ 100 GeV), and more importantly, the lightest Higgs approaches the pure CP-even composition [7,12]. It is with this figure that one arrives at the necessity of small but sufficient CP–odd composition of the lightest Higgs for saturating $(\epsilon_K)^{exp}$. Namely, CP–odd composition of the lightest Higgs boson, though small, is important for (K^0)CP constraints due to its low mass, enhancing $\langle \overline{K^0} | M_{12} | K^0 \rangle$.

Finally, in Fig. 4 we show the dependence of $\epsilon_K / (\epsilon_K)^{exp}$ on $\Delta M_K / (\Delta M_K)^{exp}$, when all the free parameters of the model vary. As the figure suggests, there are solutions in any close proximity of the experimental results.

In concluding the work, we now come back to the question mentioned in the introduction, **" Yes, there is a correlation between the Higgs and K^0 systems"**, that is,

(1) The neutral Higgs exchange is sufficient to saturate (K^0)CP with the contribution of that Higgs boson
  - which is the *lightest* of all three,
  - which is *nearly* pure *CP even*,
(2) Larger the Γ_{12}^d higher the masses of the Higgs bosons to meet $(\Delta M_K)^{exp}$,
(3) The Higgs sector should not slide to the decoupling limit, that is, the heavy and light Higgs bosons should not decouple as otherwise the CP odd composition of the lightest Higgs is washed out,

(4) The results depend only on β_K and β_d and, as long as $\beta_K \sim \beta_d$ (for saturating ΔM_{B_d}) and FCNC's in the up–quark sector are neglected, one can take the parametrization of Γ^d in (12) as an ansatze for the flavour structure with free parameters β_K and β_d, with the constraint $|\beta_{K,s,d}| < 1/2$.

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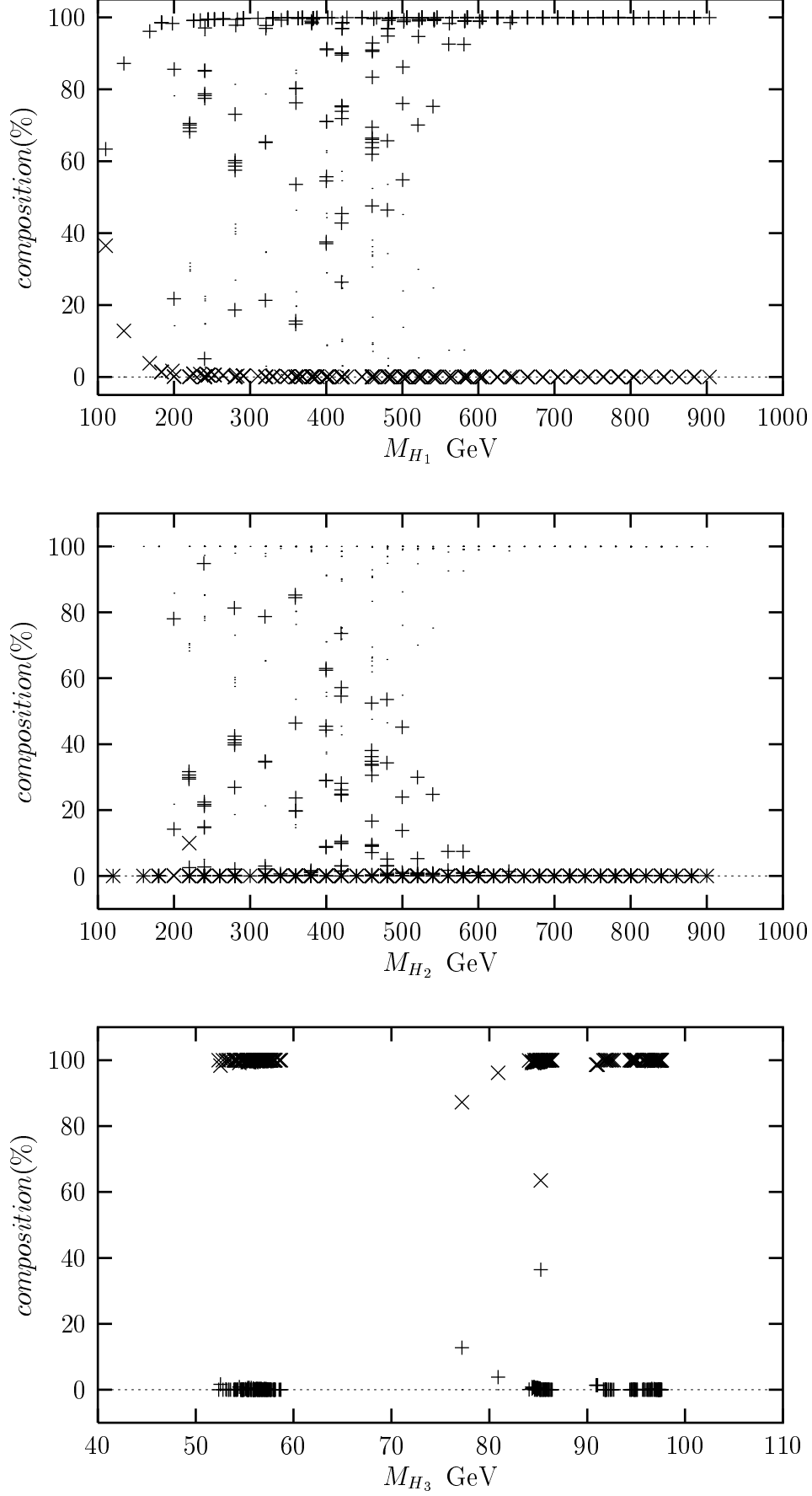[12] D. A. Demir, Phys. Lett. **B465** (1999) 177 [hep-ph/9809360].

Fig. 1. Variation of H (" $+$ "), h (" $\times$ ") and A (" $\cdot$ ") compositions (%) with their masses for H_1 (top), H_2 (middle), and H_3 (bottom). H_3 is the lightest Higgs, and
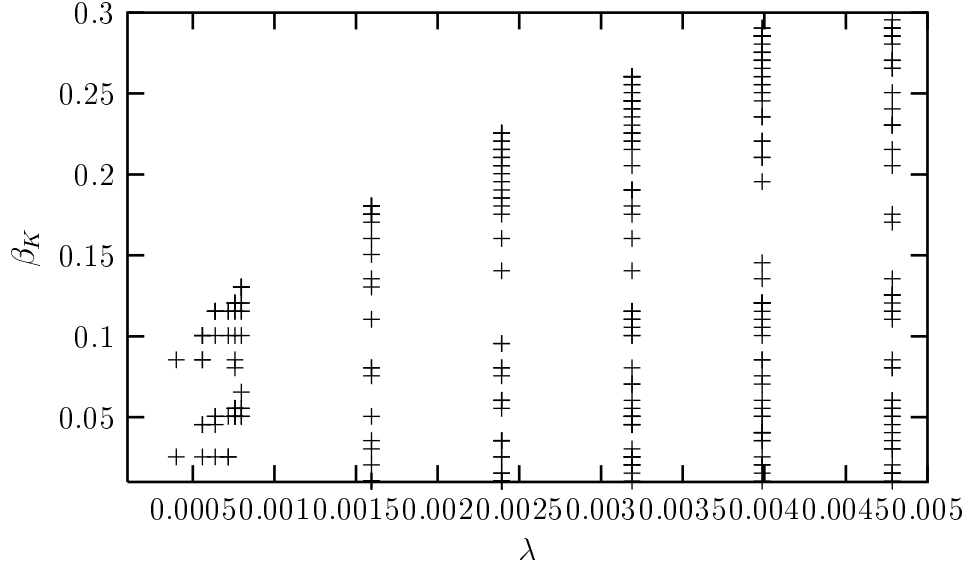
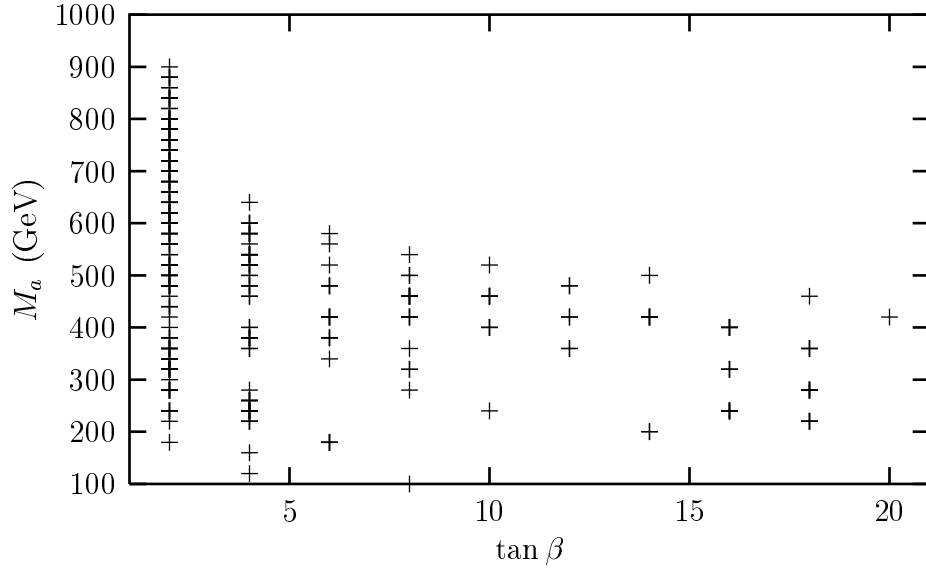Fig. 2. Variation of β_K with $\lambda \equiv |\lambda_5| = |\lambda_6| = |\lambda_7|$.
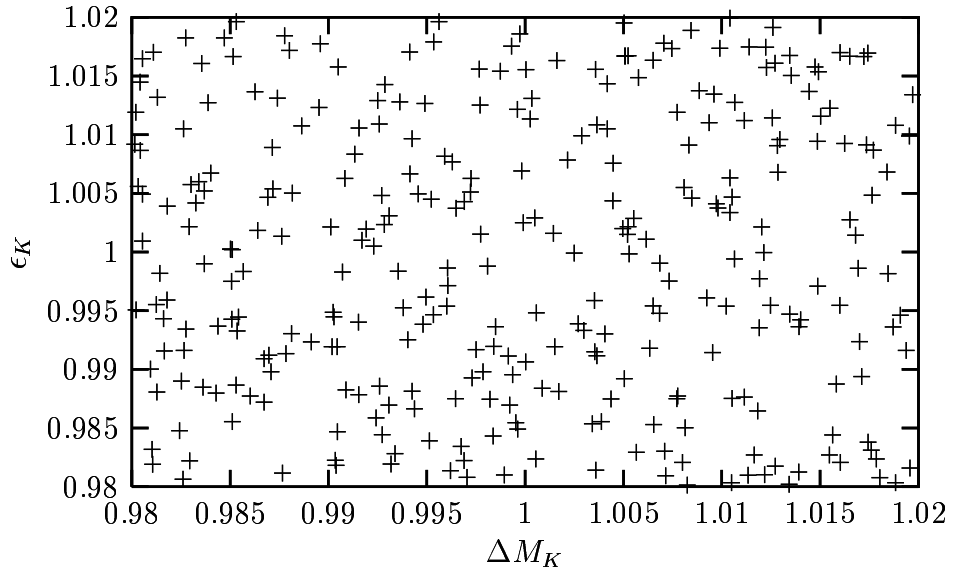

Fig. 3. Variation of M_a with $\tan\beta$

Fig. 4. Variation of ϵ_K with ΔM_K in units of their experimental values.